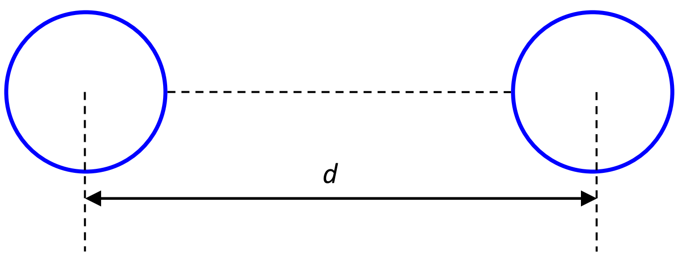
Teacher notes

Topic D

Electric potential - an unfair question from the M23 Paper 1 TZ2 exam.

Consider two identical conducting spheres of radius *R* a distance *d* apart (center to center). Each sphere has positive charge *Q* on its surface.



Where, on the horizontal dotted line, is the electric potential a maximum or a minimum?

At a distance *x* from the left center the potential is $V = \frac{kQ}{x} + \frac{kQ}{d-x} = \frac{kQd}{x(d-x)}$. The smallest value of *x* is *R* and the largest is *d* – *R*.

On the surface of each sphere the potential is $V = \frac{kQd}{R(d-R)}$.

At the midpoint ($x = \frac{d}{2}$) it is $V = \frac{4kQ}{d}$.

Which potential is greater?

To properly answer this we need calculus. What can we say without calculus?

Consider spheres with a very small radius. Then at the surface of each sphere

$$V = \frac{kQd}{R(d-R)} \approx \frac{kQd}{R(d-0)} = \frac{kQ}{R}$$

Since *R* is very small compared to *d* the potential on the spheres is greater than at the midpoint.

With calculus we find:

From $V = \frac{kQ}{x} + \frac{kQ}{d-x}$ we get

$$\frac{dV}{dx} = -\frac{kQ}{x^2} + \frac{kQ}{(d-x)^2} = kQ\frac{-(d-x)^2 + x^2}{x(d-x)^2} = kQ\frac{d(2x-d)}{x(d-x)^2}$$

Thus, if $x < \frac{d}{2}$, $\frac{dV}{dx} < 0$ and so the potential decreases as we move away from the surface of the left sphere.

And, if $x > \frac{d}{2}$, $\frac{dV}{dx} > 0$ and so the potential increases as we move past the midpoint M.

At $x = \frac{d}{2}$, $\frac{dV}{dx} = 0$ and the potential is at a minimum.

The potential looks like the following graph. Units for *V* are arbitrary.